Matrices – Q14: Eigenvectors [Problem/M](2/6/21)

For the matrix $M = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$ with eigenvalues λ_1 & λ_2, prove that $\lambda_1 + \lambda_2 = a + d$, and also that $\lambda_1\lambda_2 = |M|$

[this can be extended to 3×3 matrices]

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Solution

The characteristic equation is $\begin{vmatrix} a-\lambda & c \\ b & d-\lambda \end{vmatrix} = 0$, so that

$(a-\lambda)(d-\lambda) - bc = 0$ and $\lambda^2 - (a+d)\lambda + ad - bc = 0$

and the roots λ_1& λ_2 satisfy $\lambda_1 + \lambda_2 = a + d$ and $\lambda_1\lambda_2 = ad - bc$, as required.