Matrices – Q13: Eigenvectors [Problem/H](2/6/21)

Show that 2×2 matrices representing rotations are not diagonalisable.

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Solution

A matrix representing a rotation can be expressed in the form

$$\begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix}$$

The characteristic equation for this matrix is

$$\begin{vmatrix} cos\theta - \lambda & -sin\theta \\ sin\theta & cos\theta - \lambda \end{vmatrix} = 0$$

$$\Leftrightarrow (cos\theta - \lambda)^2 + sin^2\theta = 0$$

$$\Leftrightarrow \lambda^2 - 2cos\theta.\lambda + 1 = 0$$

The discriminant is $4cos^2\theta - 4$, which is negative for positive θ.

Thus there are no eigenvalues, and hence the matrix cannot be diagonalised.