Matrices – Q11: Eigenvectors [Problem/H](2/6/21)

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Solution

Consider $M = \begin{pmatrix} a & b \\ b & c \end{pmatrix}$, with characteristic equation

$$\begin{vmatrix} a - \lambda & b \\ b & c - \lambda \end{vmatrix} = 0$$

$$\Leftrightarrow (a - \lambda)(c - \lambda) - b^2 = 0$$

$$\Leftrightarrow \lambda^2 - (a + c)\lambda + ac - b^2 = 0$$

The discriminant is $(a + c)^2 - 4(ac - b^2) = (a - c)^2 + 4b^2$,

which is always positive, assuming that a, b & c are not all zero.

So there will be 2 distinct eigenvalues, and hence 2 linearly independent eigenvectors. Thus M is diagonalisable.