# **Maclaurin Series – Q3 [Practice/M]** (1/6/21)

Find the 1st 3 non-zero terms of the Maclaurin expansions of the following functions, and the intervals of validity of the infinite series:

(i) $\ln(3 - 2x)$

(ii) $ln\left(\frac{\sqrt{1+3x}}{1-2x}\right)$

(iii) $e^{cosx}$

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**Solution**

(i) $\ln(3-2x) = ln3\left(1-\frac{2x}{3}\right) = ln3 + \ln(1-\frac{2x}{3})$

$= ln3 + \left(-\frac{2x}{3}\right) - \frac{\left(-\frac{2x}{3}\right)^2}{2} + \cdots \approx ln3 - \frac{2x}{3} - \frac{2x^2}{9}$

The infinite series is valid for $-1 < -\frac{2x}{3} \leq 1$; ie $-\frac{3}{2} \leq x < \frac{3}{2}$

[Note: Were we to go down the route of

$\ln(3-2x) = \ln(1+[2-2x])$ , there would be an infinite number of terms involving a constant, arising from the powers of $2-2x$]

(ii) $ln\left(\frac{\sqrt{1+3x}}{1-2x}\right) = \frac{1}{2}\ln(1+3x) - \ln(1-2x)$

$= \frac{1}{2}(3x - \frac{(3x)^2}{2} + \frac{(3x)^3}{3} - \frac{(3x)^4}{4} + \cdots)$

$-([-2x] - \frac{(-2x)^2}{2} + \frac{(-2x)^3}{3} - \frac{(-2x)^4}{4} + \cdots)$

$= \frac{7x}{2} + x^2\left(-\frac{9}{4} + 2\right) + x^3\left(\frac{9}{2} + \frac{8}{3}\right) + \cdots$

$= \frac{7x}{2} - \frac{x^2}{4} + \frac{43x^3}{6} + \cdots$

valid for $x$ such that $-1 < 3x \leq 1$ and $-1 < -2x \leq 1$

ie $-\frac{1}{3} < x \leq \frac{1}{3}$ and $\frac{1}{2} > x \geq -\frac{1}{2}$

ie $-\frac{1}{3} < x \leq \frac{1}{3}$

(iii) $e^{cosx} = e^{1-\frac{x^2}{2!}+\frac{x^4}{4!}+\cdots}$

$= e\{1 + \left[-\frac{x^2}{2!} + \frac{x^4}{4!} + \cdots\right] + \frac{1}{2}\left(-\frac{x^2}{2!} + \frac{x^4}{4!} + \cdots\right)^2 + \cdots\}$

$= e\{1 - \frac{x^2}{2} + x^4\left(\frac{1}{24} + \frac{1}{8}\right) + \cdots\}$

$= e - \frac{ex^2}{2} + \frac{ex^4}{6} + \cdots$

valid for all $x$ (as both $cosx$ & $e^x$ are valid for all values).

[Note that the expansion $e^{1-\frac{x^2}{2!}+\frac{x^4}{4!}+\cdots}$

$= 1 + \left[1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots\right] + \frac{1}{2}\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots\right)^2 + \cdots$ would have involved an infinite number of constant terms (though their sum would been $e$).]