Maclaurin Series – Q2 [Practice/E] (1/6/21)

Find a Maclaurin expansion (with 3 non-zero terms) for $sin^2 x$ by two methods.

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Solution

Method 1

$$sin^2x = (x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots)(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots)$$

$$= x^2 + \frac{x^6}{(3!)^2} + 2(x)\left(-\frac{x^3}{3!}\right) + 2x\left(\frac{x^5}{5!}\right) + \cdots$$

$$= x^2 - \frac{x^4}{3} + \left(\frac{1}{36} + \frac{1}{60}\right)x^6 + \cdots$$

[though in fact the next term is negative]

$$= x^2 - \frac{x^4}{3} + \frac{8}{180}x^6 + \cdots = x^2 - \frac{x^4}{3} + \frac{2}{45}x^6 + \cdots$$

Method 2

$$sin^2x = \frac{1}{2}(1 - cos2x)$$

$$= \frac{1}{2} - \frac{1}{2}(1 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} - \frac{(2x)^6}{6!} + \cdots)$$

$$x^2 - \frac{x^4}{3} + \frac{2}{45}x^6 - \cdots$$