

## Maclaurin Series - Exercises (2 pages; 8/3/17)

(1) Find the 1st 3 non-zero terms of the Maclaurin expansions of the following functions, and the intervals of validity of the infinite series:

(i)  $\ln(3 - 2x)$

(ii)  $\ln\left(\frac{\sqrt{1+3x}}{1-2x}\right)$

(iii)  $e^{\cos x}$

(2) Find a Maclaurin expansion (with 3 non-zero terms) for  $\sin^2 x$  by two methods

(3) Expand  $\sqrt{x}$  in powers of  $x - 1$ , and investigate the accuracy of the resulting approximation for  $\sqrt{2}$  when 8 terms are taken.

(4) Find the % errors (to 2sf) associated with the following Maclaurin approximations:

(i)(a)  $\cos x = 1 - \frac{x^2}{2!}$  for  $x = \frac{\pi}{6}$  &  $x = \frac{\pi}{3}$

(b)  $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!}$  for  $x = \frac{\pi}{6}$  &  $x = \frac{\pi}{3}$

(ii)(a)  $e^x = 1 + x + \frac{x^2}{2!}$  for  $x = 1$  &  $x = 2$

(b)  $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}$  for  $x = 1$  &  $x = 2$

(iii)  $\ln(1 + x) = x - \frac{1}{2}x^2$  for  $x = 0.1, x = 0.5$  &  $x = 1$

(b)  $\ln(1 + x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4$

for  $x = 0.1$ ,  $x = 0.5$  &  $x = 1$

(5) Use the 1st 5 terms of a Maclaurin expansion to find an approximate value for  $P(Z < 1)$ , where  $Z \sim N(0,1)$  and  $Z$  has pdf

$$f(z) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}z^2\right)$$

(6) Use 3 terms of a Maclaurin expansion of  $\ln\left(\frac{1+x}{1-x}\right)$  to find an approximate value for  $\ln\left(\frac{2}{3}\right)$

(7) Find the first 3 non-zero terms, as well as the general term in the Maclaurin expansion of  $\cosh^3 x$