

MAT Problems - Miscellaneous (2 pages; 19/9/17)

[Note: These are all included in the STEP Problems]

(1) Consider the quadratic equation $x^2 + bx + c = 0$

(i) By experimenting with different examples, find conditions on b and/or c for the roots of the equation to exist and be of the same sign.

(ii) Find conditions for the roots to exist and both be positive

(2) Find the square roots of $49 - 12\sqrt{5}$

(3) Consider the sequence defined by $u_n = au_{n-1} + b$,

where a & b are real constants, and u_0 is given.

(i) What familiar sequences are special cases of this sequence?

(ii) Define a new sequence by $v_n = u_n + c$

For what value of c , in terms of a & b , will v_n be a geometric sequence?

For what value of a does this not work?

(iii) If $u_n = 2u_{n-1} + 3$, and $u_0 = 4$, find a formula for u_n in terms of n

(iv) Find a similar formula for $u_n = au_{n-1} + b$, where u_0 is given.

(v) Under what conditions will u_n be constant? Give a non-trivial example.

(4) Find the turning points of $y = (x^2 - 4x + 3)^2$

(5) Show that $\sum_{r=0}^n \binom{n}{r} = 2^n$

(6) Linear Interpolation

By approximating the graph of $y = \log_2 x$ by a straight line between $x = 2$ and $x = 4$, find an approximate value for $\log_2 \left(\frac{5}{2}\right)$

(7) (i) Does $\sqrt{4}$ equal 2 or ± 2 ? (ii) Simplify $\sqrt{x^2}$

(8) For what value of x does $(x + 2)(x + 4)$ have its minimum value?

(9) Prove that $E' \Rightarrow L'$ is equivalent to $L \Rightarrow E$

(10) Give an example of a quadratic equation that has no real roots.

(11) If $\int_{-a}^a f(x) dx = b$, find $\int_{-a}^a f(-x) dx$