

MAT Problems - Logarithms (Sol'ns) (3 pages; 19/9/17)

(1) Show that $\log(4 - \sqrt{15}) = -\log(4 + \sqrt{15})$

Solution

$$\log(4 - \sqrt{15}) = -\log\left(\frac{1}{4 - \sqrt{15}}\right) = -\log\left(\frac{4 + \sqrt{15}}{16 - 15}\right) = -\log(4 + \sqrt{15})$$

$$\begin{aligned} \text{[or } \log(4 - \sqrt{15}) + \log(4 + \sqrt{15}) &= \log\{(4 - \sqrt{15})(4 + \sqrt{15})\} \\ &= \log(16 - 15) = 0 \text{]} \end{aligned}$$

(2) If $k = \log_{24} 12$, write the following in terms of k :

(a) $\log_{24} 2$ (b) $\log_{24} 6$

Solution

$$(a) \log_{24} 2 = \log_{24} \left(\frac{24}{12}\right) = \log_{24} 24 - \log_{24} 12 = 1 - k$$

$$(b) \log_{24} 6 = \log_{24} \left(\frac{12}{2}\right) = \log_{24} 12 - \log_{24} 2 = k - (1 - k) = 2k - 1$$

$$\begin{aligned} \text{[or } \log_{24} 6 &= \log_{24} \left(\frac{24}{4}\right) = \log_{24} 24 - \log_{24} 4 = 1 - \log_{24}(2^2) \\ &= 1 - 2\log_{24} 2 = 1 - 2(1 - k) = 2k - 1 \text{]} \end{aligned}$$

(3) Write $\log_2 3$ in terms of logs to the base 10

Solution

Method 1

Standard result: $\log_a b \log_b c = \log_a c$

[a is raised to the power of $\log_a c$ in order to get to c ;
 alternatively, raise a to the power of $p = \log_a b$, to get to b , and
 then raise b to the power of $q = \log_b c$, to get to c ; thus $a^p =$
 b and $b^q = c$, which gives $(a^p)^q = c$, and hence $a^{pq} = c$, so that
 $\log_a c = pq = \log_a b \log_b c$]

Then $\log_b c = \frac{\log_{10} c}{\log_{10} b}$, so that $\log_2 3 = \frac{\log_{10} 3}{\log_{10} 2}$

Method 2

Set up an equation, as follows:

Let $\log_2 3 = x$

[The advantage of creating an equation is that we then have something that can be manipulated.]

$$\Rightarrow 3 = 2^x$$

$$\Rightarrow \log_{10} 3 = x \log_{10} 2$$

$$\Rightarrow \log_2 3 = x = \frac{\log_{10} 3}{\log_{10} 2}$$

(4) Is $\log_2 3 > \frac{3}{2}$?

Solution

$$\log_2 3 > \frac{3}{2} \Leftrightarrow 3 > 2^{\frac{3}{2}} \text{ (as } y = 2^x \text{ is an increasing function)}$$

$$\Leftrightarrow 3^2 > 2^3$$

So answer is Yes.

(5) Simplify $\frac{\log_x b}{\log_x a}$

Solution

Without loss of generality, we can suppose that $x < a < b$, with $x^p = a$

and $a^q = b$, so that $x^{pq} = b$

Then $\frac{\log_x b}{\log_x a} = \frac{pq}{p} = \log_a b$

[or $\log_x a \cdot \log_a b = \log_x b$: in terms of powers, p takes you from x to a, and q takes you from a to b; so pq takes you from x to b]