

MAT Problems E - Inequalities (2 pages; 19/9/17)

[Note: These are all included in the STEP Problems]

(1) If $f(x) \leq f(x_0)$ and $f(x) \leq a + \frac{1}{2}f(x_0)$, show that $f(x) \leq 2a$

(2) Given that $f(x)$ has a maximum on the interval $0 \leq x \leq \frac{1}{2}$ at $x = x_0$, show that $\int_0^x f(t)dt \leq \frac{1}{2}f(x_0)$ whenever $0 \leq x \leq \frac{1}{2}$

(3) Assuming that $\sin^2\theta + \cos^2\theta = 1$, but without using any compound angle results, show that $\sin\theta\cos\theta \leq \frac{1}{2}$

(4) Which is larger: $\frac{\sqrt{7}}{2}$ or $\frac{1+\sqrt{6}}{3}$ (without using a calculator)?

(5) How would you solve the inequality: $\frac{1}{x} < x$?

(6) Is $\frac{6}{7} < \frac{2}{\sqrt{5}}$?

(7) Is $\log_2 3 > \frac{3}{2}$?

(8) Are the following true or false?

(i) $a < b \Rightarrow \frac{1}{a} > \frac{1}{b}$

$$(ii) a < b \Rightarrow a^2 < b^2$$

$$(iii) a < b \ \& \ c < d \Rightarrow a + c < b + d$$

$$(iv) a < b \ \& \ c < d \Rightarrow a - c < b - d$$

(9) Prove or provide a counter-example for the conjecture

$x > a \ \& \ y > b \Rightarrow xy > ab$ (a, b real) in each of the following cases:

(i) $a > 0, b > 0$ (ii) $a < 0, b < 0$ (iii) $a > 0, b < 0$