MAT Problems C - Curve Sketching (2 pages; 19/9/17)

[Note: These are all included in the STEP Problems]

(1) Sketch the graph of $\sqrt{x^2 - 2x + 1}$ for $0 \leq x \leq 2$

(2) Suppose that $y = f(x)$ is reflected in the line $x = a$, to give $y = f(u)$. Find u in terms of x.

(3) (i) For $f(x) = ax^3 + bx^2 + cx + d$, what is the x-coordinate of the point of inflexion?

(ii) Give examples of cubic functions for which the PoI is at the Origin, and the gradient at the Origin is (a) 1 (b) -1. How do the shapes of the two graphs differ?

(4) Cubics

(i) What possible shapes might a cubic have (ignoring its position relative to the axes)?

(ii) How many stationary points does the cubic function,

$f(x) = x^3 + x^2 - 2x + 3$ have?

(iii) What is the condition for there to be 2 stationary points for the general cubic $f(x) = ax^3 + bx^2 + cx + d$?

(iv) For $f(x) = ax^3 + bx^2 + cx + d$, find the x-coordinate of any turning points of the gradient.

If the cubic has turning points, how could they be used to find the point of inflexion?

(v) For $f(x) = ax^3 + bx^2 + cx + d$, find conditions for the shape of the curve to be each of the 3 possibilities shown in (i), by considering the gradient at the point of inflexion.

(5) What happens to the graph of $y = f(x)$ when it is transformed to:

(a) $y = f(|x|)$ (b) $|y| = f(x)$

(6) What combination of transformations converts $y = 2^x$ to

$y = 2^{4x-2}$?

(7) What can be said about the graph of $f(x)$ if $(x - a)^n$ is a factor of $f(x)$, where $f(x)$ is a polynomial function and $n \in \mathbb{Z}^+$?