Notes & Solutions for Q1-5 of the Nov. 2007 MAT Paper

(4 Pages; 31/10/16)

(to be read in conjunction with the official solutions)

Q1/A

A useful question to ask is: "What can we easily do?" In this case, we can break down the expression into powers of 2 and 3.

It's only when we simplify the expression as much as possible that it becomes apparent that the problem is easily solved.

Q1/D

Drawing a diagram may indicate how to proceed with the problem. Here, for example, we see that the required point must lie on the line joining the centres of the two circles.

It is possible to find the intersection of the first circle with the line joining the two centres, but this is quite time-consuming (especially for a multiple choice question), and so it's worth looking for another method.

The official solution uses a vector approach, using the fact that the required point is $\frac{2}{5}$ of the way along the line joining the two centres, from the point (5,4).

A variation of this approach, which doesn't use vectors, is to use linear interpolation; taking a weighted average of the two centres:

$\frac{2}{5}(1,1) + \frac{3}{5}(5,4)$

Q1/H

By considering the integrals as areas under the curve, the equations can be converted into simultaneous equations in two unknowns: $A = \int_0^1 f(x)dx$ & $B = \int_1^2 f(x)dx$, with the required answer being $A + B$.

Q1/I

This is an example of a question that perhaps looks more complicated than it actually is.

The official solution says $4(log_{10}a)^2 = 1 \Rightarrow log_{10}a = \frac{1}{2}$

There is of course the possibility that $log_{10}a = -\frac{1}{2}$, but this gives a smaller value for a.

Q1/J

The given inequality may appear to be complicated, and you can't be sure initially whether any particular theoretical knowledge is required to solve the problem.

However, once we try $n = 1$ (obtaining 5150 for the left-hand side, using the sum of an arithmetic series), and note that $n = 2$ gives a larger value, then there ceases to be any problem.

Q4

For the 1st part of (ii), the official solution refers to a reflection in $y = x$. This just means that the length OP becomes the length OR and vice-versa.

Q5

(i) $f(5) = 2f(4) = 2[f(2)]^2 = 2[f(1)]^4 = 2[2f(0)]^4 = 2^5 = 32$

(ii) 4

(iii) $g(5) = 1 + g(4) = 1 + [1 + g(2)] = 2 + [1 + g(1)]$

$= 3 + [1 + g(0)] = 4$

(iv) $g(2^k) = 1 + g(0) = 1$ if $k = 0$

If $k > 0$, $g(2^k) = 1 + g(2^{k-1}) = 2 + g(2^{k-2})$

$= \cdots = k + g(2^{k-k})$

[noting that all terms are of the form $r + g(2^{k-r})$]

$= k + g(1) = k + [1 + g(0)] = k + 1$

(v) If $k = 0$, then $g(2^l + 2^k) = g(2^l + 1) = 1 + g(2^l)$

$= 1 + (l + 1) = l + 2$, from (iv)

If $k > 0$, then $g(2^l + 2^k) = g(2^k[2^{l-k} + 1])$

$= 1 + g(2^{k-1}[2^{l-k} + 1]) = \cdots = k + g(2^{l-k} + 1)$

$= k + [1 + g(2^{l-k})]$

$= k + 1 + [l - k + 1]$, from (iv)

$= l + 2$

Thus $g(2^l + 2^k) = l + 2$ in all cases.

(vi) Let RD denote the recursion depth.

From the definition of $f(n)$, for $n > 0$:

RD of $f(n) = 1 +$ RD of $f(\frac{n}{2})$

and RD of $f(n) = 1 +$ RD of $f(n-1)$

And this is the same as the definition of $g(n)$, for $n > 0$.

Also RD of $f(0) = 0$ and $g(0) = 0$