Logic: Truth Tables - Exercises (Solutions)

(7 pages; 16/7/15)

(1) Show that $(p \wedge q) \vee \sim p \vee \sim q$ is a tautology (ie is always true)

(2) Show that $p \Rightarrow q$ is equivalent to $\sim q \Rightarrow \sim p$

(3) Show that $p \Rightarrow q$ is (also) equivalent to $\sim p \vee q$

(4) Show that $p \wedge (q \vee r)$ is equivalent to $(p \wedge q) \vee (p \wedge r)$ [ie one of the distributive rules]

(5) Show that $\{p \vee (\sim q \wedge r)\} \wedge q$ is equivalent to $p \wedge q$

(6) $(a \wedge b) \vee (\sim a \wedge c) \vee (\sim b \wedge c) = \sim [(\sim a \wedge \sim c) \vee (\sim b \wedge \sim c)]$

[MEI, D2, June 2009, Q1]

(7) Show that $[(p \Rightarrow q) \wedge (\sim p \Rightarrow r)] \wedge \sim r \quad \Rightarrow q$

[MEI, D2, June2007, Q1]

Solutions

(1) Show that $(p \wedge q) \vee \sim p \vee \sim q$ is a tautology (ie is always true)

$[(p \wedge q) \vee \sim p] \vee \sim q$

p	$\wedge$	q	$\vee$	$\sim p$	$\vee$	$\sim q$
0	0	0	1	1	1	1
0	0	1	1	1	1	0
1	0	0	0	0	1	1
1	1	1	1	0	1	0

Notes

(i) In this nested truth table, the final result appears in the penultimate column.

(ii) The presence of a column of 1s for the penultimate column indicates a tautology; ie it is always true.

(iii) A separate column is sometimes given for ~ (thus, instead of A below, we could have B). This would be necessary if a column for p hadn't already been created (ie the first column, in this example).

A:

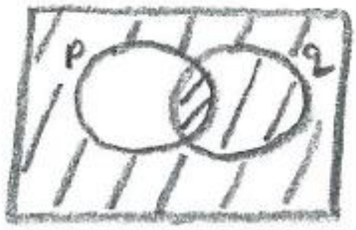
$\sim p$
1
1
0
0

B:

$\sim$	p
1	0
1	0
0	1
0	1

(iii) From the Venn diagram below, we can see that the union of

$(p \wedge q) \vee \sim p$ and $\sim q$ gives the universal set (ie the whole rectangle).



$(p \wedge q) \vee \sim p$

(2) Show that $p \Rightarrow q$ is equivalent to $\sim q \Rightarrow \sim p$

$(p$	$\Rightarrow$	$q)$	$\Leftrightarrow$	$(\sim q$	$\Rightarrow$	$\sim p)$
0	1	0	1	1	1	1
0	1	1	1	0	1	1
1	0	0	1	1	0	0
1	1	1	1	0	1	0

Notes

(i) $p \Rightarrow q$ takes the value 1 except where $p = 1$ & $q = 0$: this is the only situation that is inconsistent with $p \Rightarrow q$ (see "Logic: Implication")

(ii) $\sim q \Rightarrow \sim p$ is referred to as the contrapositive of $p \Rightarrow q$

(3) Show that $p \Rightarrow q$ is (also) equivalent to $\sim p \vee q$

(p	$\Rightarrow$	q)	$\Leftrightarrow$	($\sim p$	$\vee$	q)
0	1	0	1	1	1	0
0	1	1	1	1	1	1
1	0	0	1	0	0	0
1	1	1	1	0	1	1

(4) Show that $p \wedge (q \vee r)$ is equivalent to $(p \wedge q) \vee (p \wedge r)$ [ie one of the distributive rules]

[p	$\wedge$	(q	$\vee$	r)]	$\Leftrightarrow$	[(p	$\wedge$	q)	$\vee$	(p	$\wedge$	r)]
0	0	0	0	0	1	0	0	0	0	0	0	0
0	0	0	1	1	1	0	0	0	0	0	0	1
0	0	1	1	0	1	0	0	1	0	0	0	0
0	0	1	1	1	1	0	0	1	0	0	0	1
1	0	0	0	0	1	1	0	0	0	1	0	0
1	1	0	1	1	1	1	0	0	1	1	1	1
1	1	1	1	0	1	1	1	1	1	1	0	0
1	1	1	1	1	1	1	1	1	1	1	1	1

Note: The equivalence can also be demonstrated using a Venn diagram.

(5) Show that $\{p \vee (\sim q \wedge r)\} \wedge q$ is equivalent to $p \wedge q$

[{ p	$\vee$	($\sim$	q	$\wedge$	r)}	$\wedge$	q]	$\Leftrightarrow$	(p	$\wedge$	q)
0	0	1	0	0	0	0	0	1	0	0	0
0	1	1	0	1	1	0	0	1	0	0	0
0	0	0	1	0	0	0	1	1	0	0	1
0	0	0	1	0	1	0	1	1	0	0	1
1	1	1	0	0	0	0	0	1	1	0	0
1	1	1	0	1	1	0	0	1	1	0	0
1	1	0	1	0	0	1	1	1	1	1	1
1	1	0	1	0	1	1	1	1	1	1	1

(6) $(a \wedge b) \vee (\sim a \wedge c) \vee (\sim b \wedge c) =\sim [(\sim a \wedge \sim c) \vee (\sim b \wedge \sim c)]$

[MEI, D2, June 2009, Q1]

$[(a \wedge b) \vee (\sim a \wedge c)] \vee (\sim b \wedge c) \Leftrightarrow \sim[(\sim a \wedge \sim c) \vee (\sim b \wedge \sim c)]$

a	$\wedge$	b	$\vee$	$\sim a$	$\wedge$	c	$\vee$	$\sim b$	$\wedge$	c	$\Leftrightarrow$	$\sim$	$\sim a$	$\wedge$	$\sim c$	$\vee$	$\sim b$	$\wedge$	$\sim c$
0	0	0	0	1	0	0	0	1	0	0	1	0	1	1	1	1	1	1	1
0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0	1	0	0
0	0	1	0	1	0	0	0	0	0	0	1	0	1	1	1	1	0	0	1
0	0	1	1	1	1	1	1	0	0	1	1	1	1	0	0	0	0	0	0
1	0	0	0	0	0	0	0	1	0	0	1	0	0	0	1	1	1	1	1
1	0	0	0	0	0	1	1	1	1	1	1	1	0	0	0	0	1	0	0
1	1	1	1	0	0	0	1	0	0	0	1	1	0	0	1	0	0	0	1
1	1	1	1	0	0	1	1	0	0	1	1	1	0	0	0	0	0	0	0

(7) Show that $[(p \Rightarrow q) \wedge (\sim p \Rightarrow r)] \wedge \sim r \quad \Rightarrow q$

[MEI, D2, June2007, Q1]

$$\{[(p \Rightarrow q) \wedge (\sim p \Rightarrow r)] \wedge \sim r\} \Rightarrow q$$

p	$\Rightarrow$	q	$\wedge$	$\sim p$	$\Rightarrow$	r	$\wedge$	$\sim r$	$\Rightarrow$	q
0	1	0	0	1	0	0	0	1	1	0
0	1	0	1	1	1	1	0	0	1	0
0	1	1	0	1	0	0	0	1	1	1
0	1	1	1	1	1	1	0	0	1	1
1	0	0	0	0	1	0	0	1	1	0
1	0	0	0	0	1	1	0	0	1	0
1	1	1	1	0	1	0	1	1	1	1
1	1	1	1	0	1	1	0	0	1	1