Logarithms (5 pages; 5/6/23)

See also Logarithms (STEP).

Contents

(1) Converting an exponential equation into a logarithmic equation

Example 1: $2^x = 9$

The equivalent logarithmic equation takes the form $log_a b = c$

where a is the base (2) and c is the power (x).

This is how the logarithm is defined, and it can be seen to be the power in the exponential equation.

So $log_2 9 = x$

Obviously the reverse process could be carried out:

Example 2: $log_2 8 = x$

The base is 2 and the power is x.

So $2^x = 8$, and hence $x = 3$ (the exponential equation only helps because 8 is a simple power of 2).

(2) Logarithm function as the inverse of the exponential function

If $y = a^x$, then $log_a y = x$

Thus, x is mapped to y by the rule $y = a^x$,

and y is mapped to x by the rule $x = log_a y$

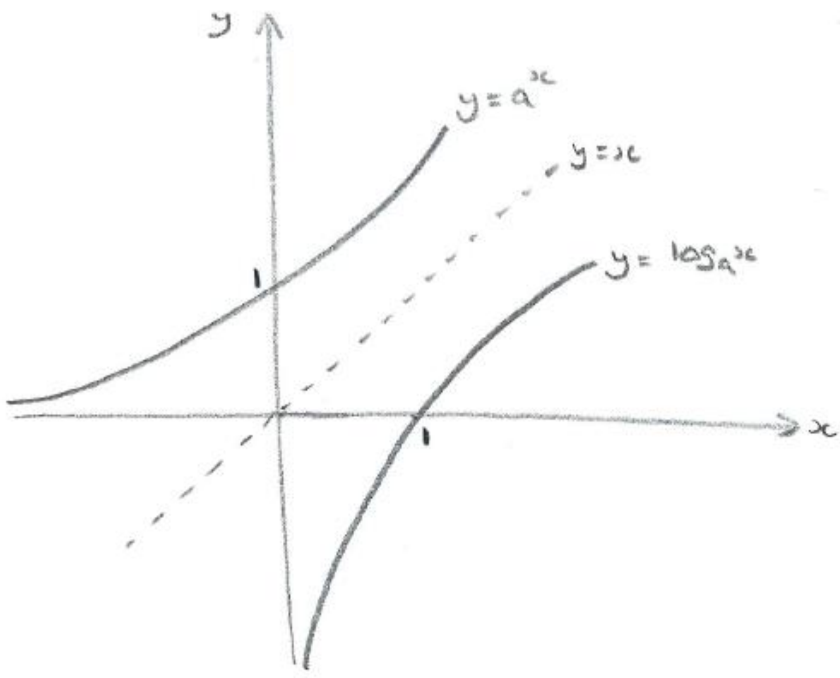
Then, in order for this to be expressed as a function with x (rather than y) on the horizontal axis, we write $y = log_a x$

Thus the logarithm function is the opposite of the exponential function, and vice versa. Each is the 'inverse function' of the other.

[Note: $y = a^x$ is referred to here as the (general) exponential function, but "**the** exponential function" is often taken to mean $y = e^x$, with other exponential functions described as "**an** exponential function".]

The graphs of inverse functions are obtained from each other by reflection in the line $y = x$. In this case, $y = a^x$ and $x = log_a y$ represent the same graph (as we have only made a rearrangement of the equation), but by converting $x = log_a y$

to $y = log_a x$ we are swapping the roles of x & y, which is equivalent to a reflection in the line $y = x$. See diagram below.



As the logarithmic function is the inverse of the exponential function, and vice versa, the operations of raising to a power and taking the logarithm can be thought of as cancelling each other out, so that $log_a(a^x) = x$ and $a^{log_a x} = x$

(3) Taking the logarithm of both sides of an equation

Example: A population is modelled by the equation $P = a.2^{bt}$

Option 1: Take logs to base 2

$log_2P = log_2a + log_2(2^{bt}) = log_2a + bt$

(so that if log_2P is plotted against t, a straight line pattern should emerge, if the model is correct; the line has a gradient of b and

'y-intercept' of log_2a)

Option 2: Take logs to base 10 (eg if logs to base 2 can't be calculated)

$log_{10}P = log_{10}a + log_{10}(2^{bt}) = log_{10}a + bt.log_{10}2$

(which, again, is the equation of a straight line).

(4) Change of base

$log_ab\ log_bc = log_ac$ or $log_bc = \frac{log_ac}{log_ab}$

Proof

Let $b = a^x$ & $c = b^y$

Then $c = (a^x)^y = a^{xy}$

and $log_ac = xy = log_ab\ log_bc$

Special case: $log_bc = \frac{1}{log_cb}$

(5) Manipulating logarithms

eg $3 + 2log_25 = 3log_22 + log_2(5^2)$

$= log_2(2^3) + log_2(5^2) = log_2(8 \times 25) = log_2(200)$

Appendix: Useful results

(1) $log_a b = c \Leftrightarrow a^c = b$

(2) $log_a b\ log_b c = log_a c$ or $log_b c = \frac{log_a c}{log_a b}$

Special case: $log_b c = \frac{1}{log_c b}$