Linear Programming Exercise (Solution)

(3 pages; 23/1/2014)

Maximise $\mathrm{P} = 2x + 3y$

subject to $x + 2y \leq 12$

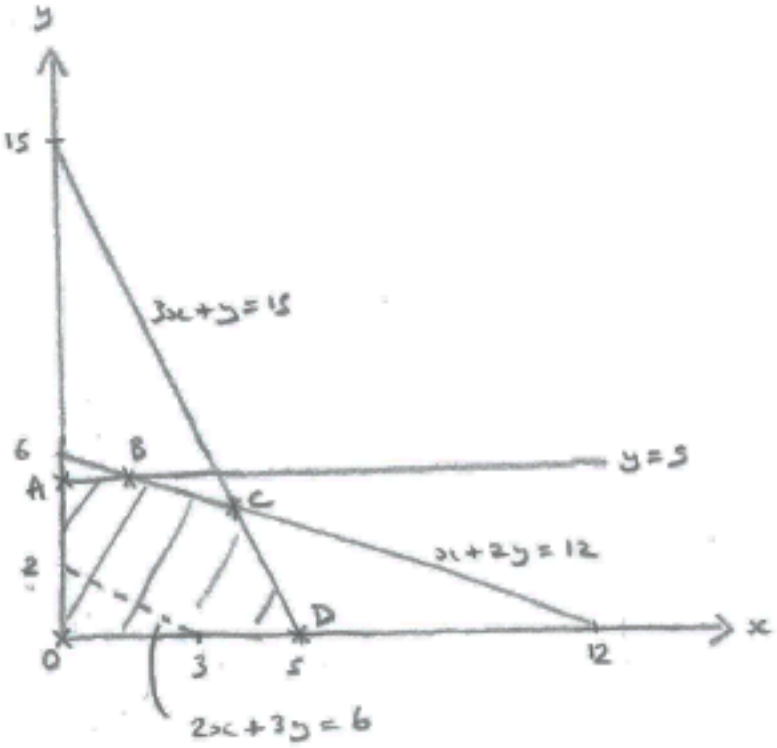
$3x + y \leq 15$

$y \leq 5$

$x \geq 0\ ,\ y \geq 0$

Integer values of x & y are required.

Solution



By considering lines parallel to $2x + 3y = 6$ (for example), C is the optimal solution, if x& y are allowed to take non-integer values.

At C:

$x + 2y = 12$ (1)

$3x + y = 15$ (2)

$x + 2y = 12$ (1)

$6x + 2y = 30$ (3)= 2×(2)

$(3) - (1) \Rightarrow 5x = 18 \Rightarrow x = \frac{18}{5} = 3.6$

Then $(2) \Rightarrow y = 15 - \frac{54}{5} = \frac{21}{5} = 4.2$

$\Rightarrow P = 7.2 + 12.6 = 19.8$

Check:

At A, P=15

At B, P=19

At D, P=10

(confirming that C is the optimal vertex).

To find integer solutions, consider neighbouring points:

(3,4): $x + 2y = 11, 3x + y = 13,\ P = 18$

(3,5): $x + 2y = 13$ (reject)

(4,4): $x + 2y = 12, 3x + y = 16$ (reject)

(4,5): $x + 2y = 14$ (reject)

Thus a good solution is $x = 3, y = 4$, when $P = 18$.

However, note that if we consider the point (2,5):

$x + 2y = 12, 3x + y = 11,\ P = 19$

Thus, we cannot guarantee to find the optimal solution by the above method.