Linear Programming (23/1/2014)

Example 1

maximise $\mathrm{P} = x + y$ (eg total number of cakes made)

subject to $2x + 3y \leq 12$ (1st constraint on cost of ingredients)

$6x + 5y \leq 30$ (2nd constraint on cost of ingredients)

$x \geq 0$, $y \geq 0$

x & y are **control variables**

$\mathrm{P} = x + y$ is the **objective function**

inequalities are **(linear) constraints**

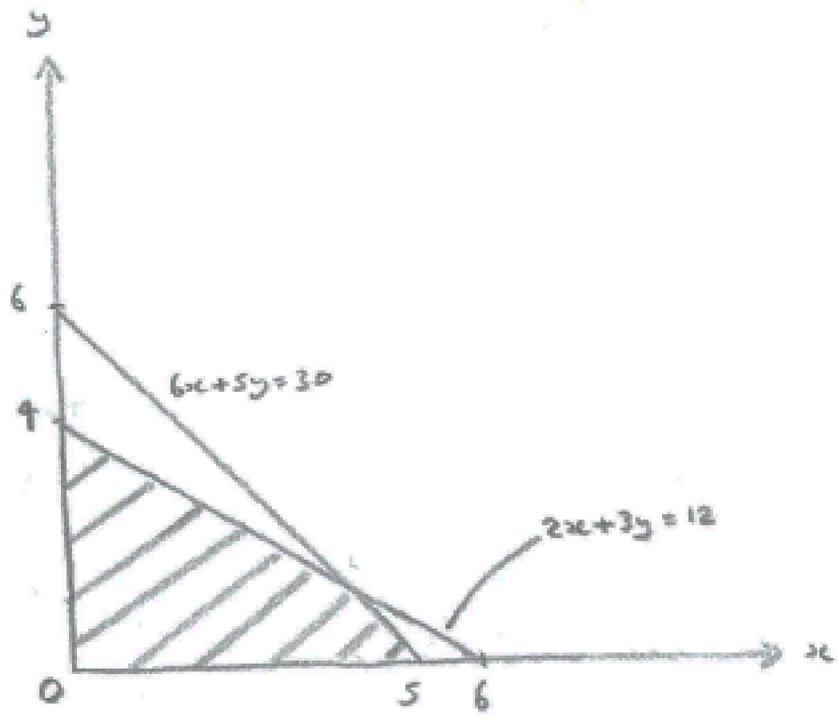

Figure 1

The shaded area in Figure 1 is the **feasible region** (ie where all the constraints are satisfied.

In more complicated cases, the unwanted areas can be shaded, leaving the feasible region as the area with no shading.

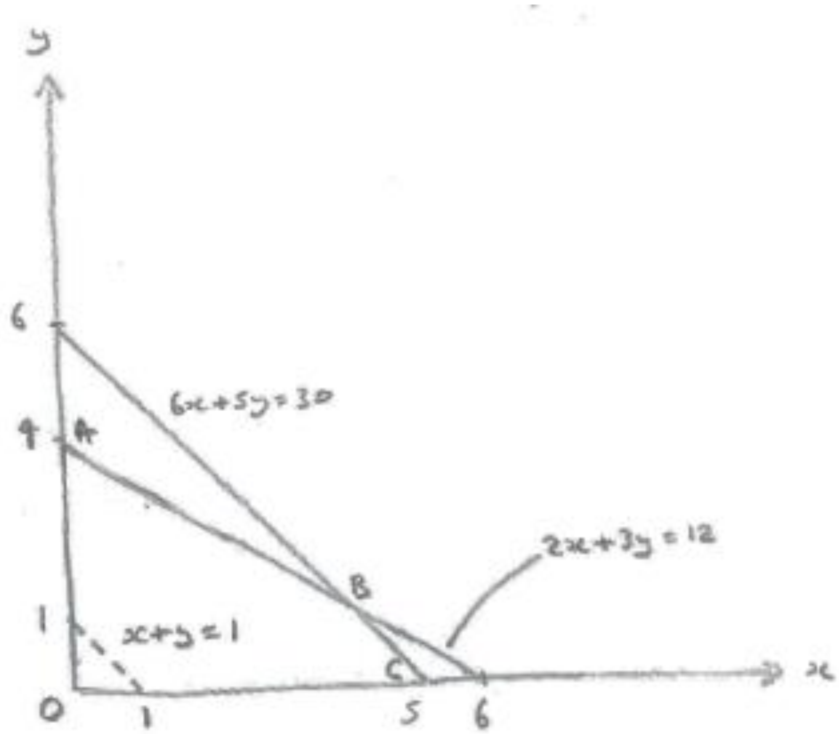

Figure 2

Referring to Figure 2, the line $P = x + y$ will be parallel to $x + y = 1$, and as far away from O as possible.

Unless the line $P = x + y$ has the same gradient as one of the constraint lines, the maximum value of P will occur at one of the vertices of the feasible region. In this example it is B.

If it isn't easy to tell from your sketch, you could simply evaluate the objective function at each vertex.

At B, $6x + 5y = 30$ (1)

& $2x + 3y = 12$ (2)

$6x + 5y = 30$ (1)

$6x + 9y = 36$ (2)

$(2) - (1) \Rightarrow 4y = 6 \Rightarrow y = \frac{3}{2} = 1.5$

$(2) \Rightarrow 2x = 12 - \frac{9}{2} \Rightarrow x = 6 - \frac{9}{4} = \frac{15}{4} = 3.75$

$\Rightarrow P = \frac{21}{4} = 5.25$ at (3.75,1.5)

At A, $P = 4$; at C, $P = 5$; confirming that B is the required vertex.

If only integer values are acceptable (as in this example, where x & y represent numbers of cakes), we can consider points neighbouring (3.75, 1.5), provided they are within the feasible region.

We require $6x + 5y \leq 30$ & $2x + 3y \leq 12$

(3,1): $6x + 5y = 23$ & $2x + 3y = 9$; $P = 4$

(3,2): $6x + 5y = 28$ & $2x + 3y = 12$; $P = 5$

(4,1): $6x + 5y = 29$ & $2x + 3y = 11$; $P = 5$

(4,2): $6x + 5y = 34$ (reject)

Thus the points (3,2) and (4,1) give an equally good solution.

However, there is no guarantee that this will be the optimal solution.

Example 2

minimise $\mathrm{P} = 2x + y$

subject to $y \geq 3$

$3x + 4y \geq 24$

$y \leq 3x$

$x \geq 0$, $y \geq 0$

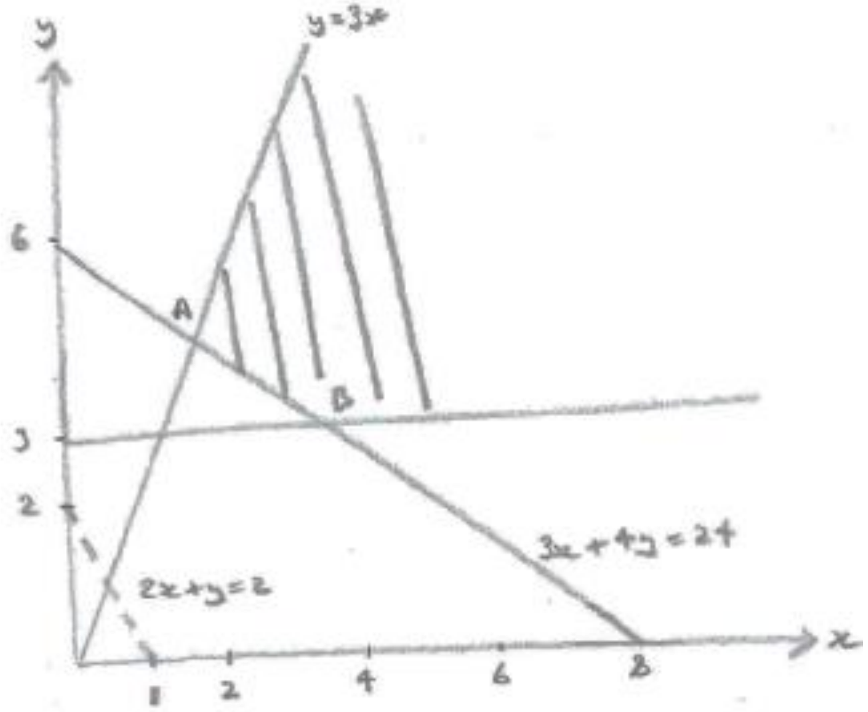

Figure 3

By considering lines parallel to $2x + y = 2$, we can see that P is minimised at A.

$3x + 4y = 24$ & $y = 3x \Rightarrow 15x = 24 \Rightarrow x = \frac{8}{5} = 1.6$; thus A is (1.6,4.8), where P = 8