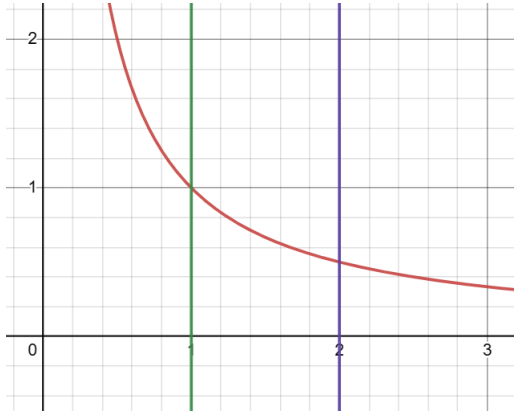


## Integration – Q8: Volume of Revolution (21/11/23)

The region bounded by the curve  $y = \frac{1}{x}$ , the lines  $x = 1$ ,  $x = 2$ , and the  $x$ -axis is rotated about the  $y$ -axis through  $360^\circ$ . Find the volume generated.



**Solution**

The region can be divided into two parts: the rectangle at the bottom, with base 1 and height  $\frac{1}{2}$  (A), and the remainder (B).

$$\begin{aligned}\text{Volume} &= \pi(2^2 - 1^2) \left(\frac{1}{2}\right) + \pi \int_{\frac{1}{2}}^1 x^2 - 1^2 dy \\ &= \frac{3\pi}{2} + \pi \int_{\frac{1}{2}}^1 y^{-2} - 1^2 dy \\ &= \frac{3\pi}{2} + \pi \left[ -y^{-1} - y \right]_{\frac{1}{2}}^1 \\ &= \frac{3\pi}{2} + \pi \left( -2 - \left[ -\frac{5}{2} \right] \right) \\ &= 2\pi\end{aligned}$$