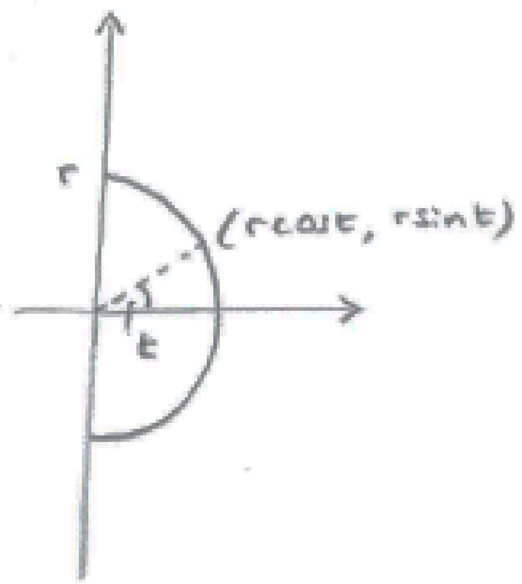
Integration – Q7: Surface Area (21/11/23)

Use integration with respect to a suitable parameter to show that the surface area of a sphere of radius r is $4\pi r^2$.

Solution

Consider the hemisphere obtained by rotating a quarter circle about the x-axis, as in the diagram below.



Surface area of hemisphere $= \int_{t_1}^{t_2} 2\pi y\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\ dt$

$= \int_0^{\frac{\pi}{2}} 2\pi(rsint)\sqrt{(-rsint)^2 + (rcost)^2}\ \mathrm{dt}$

$= 2\pi r^2 \int_0^{\frac{\pi}{2}} sint\ dt$

$= 2\pi r^2[-cost]_0^{\frac{\pi}{2}}$

$= 2\pi r^2(0 - (-1))$

$= 2\pi r^2$

Then the surface area of the sphere is double this, to give $4\pi r^2$.