Integration Overview (20/11/23)

General

Q2 [Problem/M]

Explain the following 'paradox':

$\int \frac{1}{2x} dx = \frac{1}{2}\int \frac{1}{x} dx = \frac{1}{2} lnx + C$

but $\int \frac{1}{2x} dx = \frac{1}{2}\ln(2x) + C$ (by the reverse Chain rule)

Q5 [Problem/H]

Given that $\int \frac{1}{x} dx = \ln x$ for $x > 0$, show that $\int \frac{1}{x} dx = ln|x|$ for all $x \neq 0$

Improper Integrals

Q1

Find the values of the following integrals, or show that they are not defined.

(i) $\int_{-2}^{\infty} \frac{1}{x^2} dx$

(ii) $\int_{-\infty}^{-\frac{1}{2}} e^{2x}\, dx$

(iii) $\int_{0}^{1} x^{-\frac{2}{3}}\, dx$

Arc Length

Q6

The curve C has equation $y = \frac{1}{3}x^3 + \frac{1}{4x}$. The points A and B on C have x coordinates 1 and 2, respectively. Find the length of the arc from A to B.

Volume of Revolution / Surface Area

Q3

The region between the line $y = 6 - 2x$ and the curve $y = \frac{4}{x}$ is rotated about the y-axis through 360°. Find the exact volume generated.

Q4

The region between the parabola $y^2 = 4x$, the x-axis and the line $x = 1$ is rotated about the x-axis through 360°.

(i) Find the exact volume generated:

(a) by integrating with respect to x

(b) by integrating with respect to the parameter t, where $x = t^2$ and $y = 2t$

(ii) Use the mean value of the function to carry out a rough check on your answer in (i).

(iii) Find the curved surface area associated with the volume generated in (i):

(a) by integrating with respect to x

(b) by integrating with respect to y

(c) by integrating with respect to t

Q7

Use integration with respect to a suitable parameter to show that the surface area of a sphere of radius r is $4\pi r^2$.

Q8

The region bounded by the curve $y = \frac{1}{x}$, the lines $x = 1, x = 2$, and the x-axis is rotated about the y-axis through 360°. Find the volume generated.