Integration - Exercises - Part 4 (Misc) (2 pages; 8/8/19)

(1) If $\int_{-a}^{a} f(x)\,dx = b$, find $\int_{-a}^{a} f(-x)\,dx$

(2) Explain the following 'paradox':

$\int \frac{1}{2x}dx = \frac{1}{2}\int \frac{1}{x}dx = \frac{1}{2}lnx + C$

but $\int \frac{1}{2x}dx = \frac{1}{2}\ln(2x) + C$ (by the reverse Chain rule)

(3) The region between the line $y = 6 - 2x$ and the curve $y = \frac{4}{x}$ is rotated about the y-axis through 360°. Find the exact volume generated.

(4) The region between the parabola $y^2 = 4x$, the x-axis and the line $x = 1$ is rotated about the x-axis through 360°.

(i) Find the exact volume generated:

(a) by integrating with respect to x

(b) by integrating with respect to the parameter t, where $x = t^2$ and $y = 2t$

(ii) Use the mean value of the function to carry out a rough check on your answer in (i).

(iii) Find the curved surface area associated with the volume generated in (i):

(a) by integrating with respect to x

(b) by integrating with respect to y

(c) by integrating with respect to t

(5) The curve C has equation $y = \frac{1}{3}x^3 + \frac{1}{4x}$. The points A and B on C have x coordinates 1 and 2, respectively. Find the length of the arc from A to B.

(6) Use integration with respect to a suitable parameter to show that the surface area of a sphere of radius r is $4\pi r^2$.