Inequalities - Exercises (2 pages; 21/2/20)

Key to difficulty:

* easier

** moderate

*** harder

(1*) How would you solve the inequality: $\frac{1}{x} < x$?

(2*) Is $\frac{6}{7} < \frac{2}{\sqrt{5}}$?

(3*) Which is larger: $\frac{\sqrt{7}}{2}$ or $\frac{1+\sqrt{6}}{3}$ (without using a calculator)?

(4*) Show that $e^3 > 4e^{\frac{3}{2}}$

(5*) Are the following true or false?

(i) $a < b \Rightarrow \frac{1}{a} > \frac{1}{b}$

(ii) $a < b \Rightarrow a^2 < b^2$

(iii) $a < b \,\&\, c < d \Rightarrow a + c < b + d$

(iv) $a < b \,\&\, c < d \Rightarrow a - c < b - d$

(6*) Prove or provide a counter-example for the conjecture

$x > a \,\&\, y > b \Rightarrow xy > ab \quad (a, b\ real)$ in each of the following cases:

(i) $a > 0, b > 0$ (ii) $a < 0, b < 0$ (iii) $a > 0, b < 0$

(7*) Prove that $a + b < 1 + ab$ if $a > 1\ and\ b > 1$

(8***) Solve the following inequality

$\frac{x}{x-1} \leq \frac{3}{x+2}$ $(x \neq 1, x \neq -2)$

(9***) Solve the following inequality

$|x^2 - 3| > 3x + 1$