

**Inequalities – Q4 [Practice/M] (21/6/23)**

Solve  $\frac{x-2}{x-1} < 3$

**Solution****Method 1 ('Case by case')**

$$\text{Case 1: } x - 1 > 0$$

$$\Rightarrow x - 2 < 3x - 3$$

$$\Rightarrow 1 < 2x \Rightarrow x > \frac{1}{2}$$

$$x - 1 > 0 \ \& \ x > \frac{1}{2} \Rightarrow x > 1$$

$$\text{Case 2: } x - 1 < 0$$

$$\Rightarrow x - 2 > 3x - 3$$

$$\Rightarrow 1 > 2x \Rightarrow x < \frac{1}{2}$$

$$x - 1 < 0 \ \& \ x < \frac{1}{2} \Rightarrow x < \frac{1}{2}$$

$$\text{Overall solution: } x < \frac{1}{2} \text{ or } x > 1$$

**Method 2: Multiplying by the square of the denominator**

$$(x - 2)(x - 1) < 3(x - 1)^2$$

$$\Rightarrow (x - 1)(x - 2 - 3(x - 1)) < 0$$

$$\Rightarrow (x - 1)(-2x + 1) < 0$$

$$\Rightarrow (x - 1)(2x - 1) > 0$$

$$\Rightarrow x > 1 \ \& \ x > \frac{1}{2} \quad \text{or} \quad x < 1 \ \& \ x < \frac{1}{2}$$

$$\text{ie } x > 1 \text{ or } x < \frac{1}{2}$$

**Method 3: Rearranging into the form  $\frac{f(x)}{g(x)} < 0$  etc**

$$\frac{x-2}{x-1} - \frac{3(x-1)}{x-1} < 0$$

$$\Rightarrow \frac{-2x+1}{x-1} < 0$$

$$\Rightarrow \frac{2x-1}{x-1} > 0$$

critical values of  $x$ :  $\frac{1}{2}$  and 1

*eg*  $x = 0$ :  $LHS = 1 > 0$

*eg*  $x = \frac{3}{4}$ :  $LHS = \frac{\left(\frac{1}{2}\right)}{\left(-\frac{1}{4}\right)} = -2 < 0$

*eg*  $x = 100$ :  $LHS > 0$



Solution:  $x < \frac{1}{2}$  or  $x > 1$

**Method 4 (Graph)**

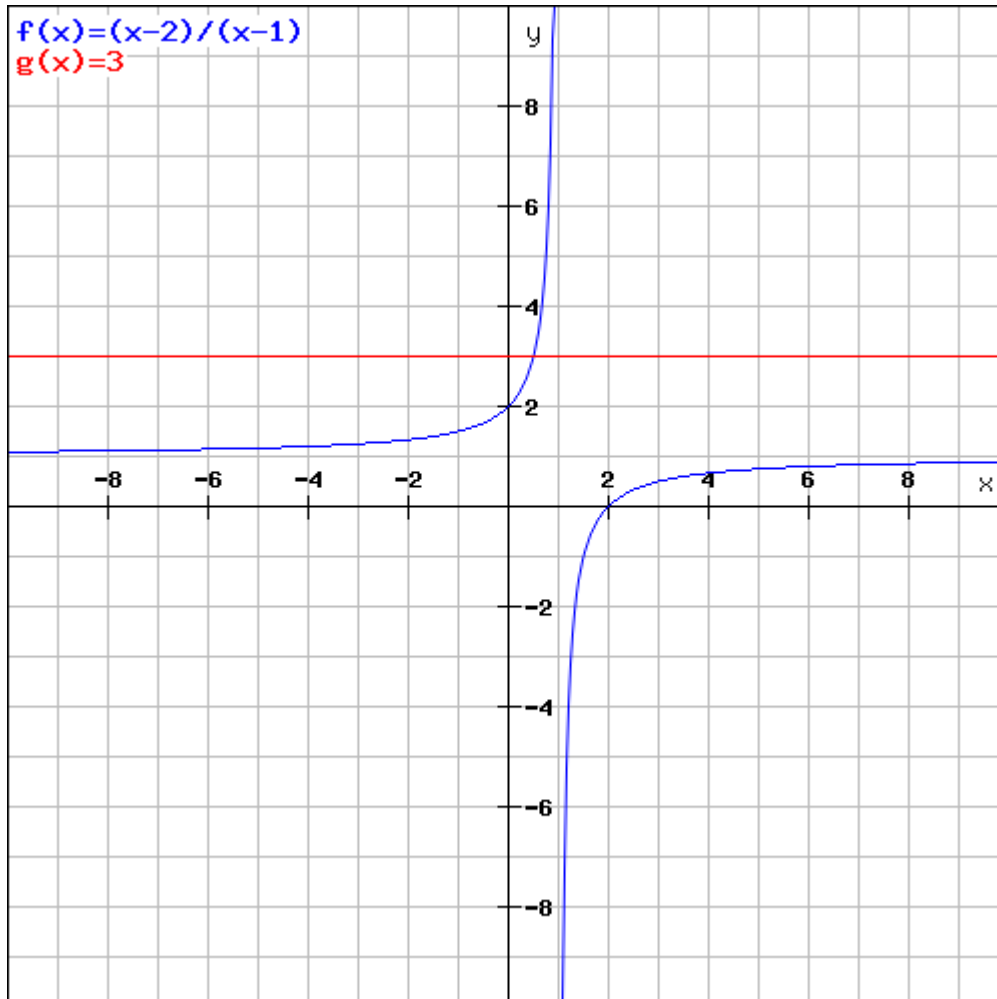
Consider the intersection of  $y = \frac{x-2}{x-1}$  and  $y = 3$ :

$$\frac{x-2}{x-1} = 3$$

$$\Rightarrow x - 2 = 3x - 3$$

$$\Rightarrow 1 = 2x \Rightarrow x = \frac{1}{2}$$

$y = \frac{x-2}{x-1}$  can be sketched by noting the vertical asymptote of  $x = 1$  (with  $y > 0$  for  $x = 1 - \delta$ ), and the horizontal asymptote of  $y = 1$  (with  $y < 1$  for eg  $x = 100$ )



[Noting that  $\frac{x-2}{x-1} = 1 - \frac{1}{x-1}$ , the graph could also be obtained from  $y = \frac{1}{x}$  by the following sequence of transformations:

- (i) translation of  $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ , to give  $y = \frac{1}{x-1}$
- (ii) reflection in the  $x$  axis, to give  $y = -\frac{1}{x-1}$
- (iii) translation of  $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ , to give  $y = 1 - \frac{1}{x-1}$

Then from the graph,  $\frac{x-2}{x-1} < 3$  when  $x < \frac{1}{2}$  or  $x > 1$