Inequalities – Q3 [Practice/H] (21/6/23)

Solve the inequality $|x^2 - 3| > 3x + 1$

Solution



To determine the points of intersection of $y = |x^2 - 3|$ and y = 3x + 1: $x^2 - 3 \ge 0$ and $x^2 - 3 = 3x + 1 \implies x^2 - 3x - 4 = 0$ $\implies (x - 4)(x + 1) = 0$

From the graph, we can reject the negative value of x, to give x = 4, which is consistent with $x^2 - 3 \ge 0$.

Also,
$$x^2 - 3 < 0$$
 and $-(x^2 - 3) = 3x + 1 \Rightarrow x^2 + 3x - 2 = 0$
 $\Rightarrow x = \frac{-3 \pm \sqrt{9+8}}{2}$

From the graph, we can reject the negative value of x, to give $\frac{-3+\sqrt{17}}{2}$, which is consistent with $x^2 - 3 < 0$.

So, from the graph, $|x^2 - 3| > 3x + 1$ when $x < \frac{-3 + \sqrt{17}}{2}$ or x > 4