

Inequalities – Q2 [Practice/M] (21/6/23)

Solve the following inequality $\frac{x}{x-1} \leq \frac{3}{x+2}$ ($x \neq 1, x \neq -2$)

Solution**Method 1**

$$\frac{x}{x-1} \leq \frac{3}{x+2}$$

Multiply both sides by $(x-1)^2(x+2)^2$ [as this will be positive]:

$$x(x-1)(x+2)^2 \leq 3(x-1)^2(x+2)$$

$$\Rightarrow (x-1)(x+2)\{x(x+2) - 3(x-1)\} \leq 0$$

$$\Rightarrow (x-1)(x+2)(x^2 - x + 3) \leq 0$$

As $x^2 - x + 3 = \left(x - \frac{1}{2}\right)^2 - \frac{1}{4} + 3 > 0$ for all x ,

the original inequality is satisfied when $-2 \leq x \leq 1$ [for example, by considering the graph of $y = (x-1)(x+2)$].

Method 2

$$\frac{x}{x-1} \leq \frac{3}{x+2} \Rightarrow \frac{x}{x-1} - \frac{3}{x+2} \leq 0$$

$$\Rightarrow \frac{x(x+2) - 3(x-1)}{(x-1)(x+2)} \leq 0 \Rightarrow \frac{x^2 - x + 3}{(x-1)(x+2)} \leq 0$$

As before, $x^2 - x + 3 > 0$ for all x ,

so that we require $(x-1)(x+2) \leq 0$, and hence $-2 \leq x \leq 1$