Induction – Q9 [Practice/E] (18/6/23)

$\sum_{r=1}^{n} \frac{1}{(2r-1)(2r+1)} = \frac{n}{2n+1}$

Solution

[Show that the result is true for $n = 1$]

Now assume that the result is true for $n = k$, so that

$\sum_{r=1}^{k} \frac{1}{(2r-1)(2r+1)} = \frac{k}{2k+1}$

The target result is $\sum_{r=1}^{k+1} \frac{1}{(2r-1)(2r+1)} = \frac{k+1}{2k+3}$

LHS $= \frac{k}{2k+1} + \frac{1}{(2k+1)(2k+3)}$

$= \frac{k(2k+3)+1}{(2k+1)(2k+3)} = \frac{2k^2+3k+1}{(2k+1)(2k+3)} = \frac{(2k+1)(k+1)}{(2k+1)(2k+3)} = \frac{k+1}{2k+3}$

[Standard wording]