

**Induction – Q6 [Practice/E] (18/6/23)**

$$\sum_{r=1}^n r(r+2) = \frac{1}{6}n(n+1)(2n+7)$$

**Solution**

[Show that the result is true for  $n = 1$ ]

Now assume that the result is true for  $n = k$ , so that

$$\sum_{r=1}^k r(r+2) = \frac{1}{6}k(k+1)(2k+7)$$

$$\text{Then } \sum_{r=1}^{k+1} r(r+2) = \frac{1}{6}k(k+1)(2k+7) + (k+1)(k+3)$$

$$= \frac{1}{6}(k+1)\{k(2k+7) + 6(k+3)\}$$

$$= \frac{1}{6}(k+1)(2k^2 + 13k + 18)$$

$$= \frac{1}{6}(k+1)(2k+9)(k+2)$$

$$= \frac{1}{6}(k+1)([k+1]+1)(2[k+1]+7)$$

[Standard wording]