Induction – Q5 [Practice/E] (18/6/23)

$$\sum_{r=1}^{n} r(r+1) = \frac{1}{3}n(n+1)(n+2)$$

Solution

[Show that the result is true for $n = 1$]

Now assume that the result is true for $n = k$, so that

$\sum_{r=1}^{k} r(r + 1) = \frac{1}{3}k(k + 1)(k + 2)$

Then $\sum_{r=1}^{k+1} r(r + 1) = \frac{1}{3}k(k + 1)(k + 2) + (k + 1)(k + 2)$

$= \frac{1}{3}(k + 1)(k + 2)(k + 3)$

$= \frac{1}{3}(k + 1)([k + 1] + 1)([k + 1] + 2)$

[Standard wording]