Induction – Q3 [Practice/E] (18/6/23)

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots+\frac{1}{2^n}=1-\frac{1}{2^n}$$

Solution

Result to prove: $\sum_{r=1}^{n} \frac{1}{2^r} = 1 - \frac{1}{2^n}$

[Show that the result is true for $n = 1$]

Now assume that the result is true for $n = k$, so that

$\sum_{r=1}^{k} \frac{1}{2^r} = 1 - \frac{1}{2^k}$

The target result is $\sum_{r=1}^{k+1} \frac{1}{2^r} = 1 - \frac{1}{2^{k+1}}$

Then $\sum_{r=1}^{k+1} \frac{1}{2^r} = 1 - \frac{1}{2^k} + \frac{1}{2^{k+1}}$

$= 1 - \frac{1}{2^{k+1}}(2 - 1) = 1 - \frac{1}{2^{k+1}}$, which is the target.

[Standard wording]