Induction – Q31 [Practice/E] (18/6/23)

If $A = \begin{pmatrix} -1 & -4 \\ 1 & 3 \end{pmatrix}$, then $A^n = \begin{pmatrix} 1 - 2n & -4n \\ n & 1 + 2n \end{pmatrix}$

Solution

[Show that the result is true for $n = 1$]

Now assume that the result is true for $n = k$

so that $A^k = \begin{pmatrix} 1-2k & -4k \\ k & 1+2k \end{pmatrix}$

Target is: $A^{k+1} = \begin{pmatrix} -1-2k & -4k-4 \\ k+1 & 3+2k \end{pmatrix}$

$A^{k+1} = \begin{pmatrix} -1 & -4 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 1-2k & -4k \\ k & 1+2k \end{pmatrix}$

$= \begin{pmatrix} -1+2k-4k & 4k-4-8k \\ 1-2k+3k & -4k+3+6k \end{pmatrix}$

$= \begin{pmatrix} -1-2k & -4k-4 \\ k+1 & 3+2k \end{pmatrix}$

[Standard wording]