Induction – Q28 [Practice/M] (18/6/23)

$\sum_{r=1}^{n} r^2 > \frac{1}{3}n^3$

Solution

[Show that the result is true for $n = 1$]

Now assume that the result is true for $n = k$

so that $\sum_{r=1}^{k} r^2 > \frac{1}{3}k^3$

Then $\sum_{r=1}^{k+1} r^2 > \frac{1}{3}k^3 + (k+1)^2$

$= \frac{1}{3}(k+1)^3 - \frac{1}{3}(3k^2 + 3k + 1) + (k+1)^2$

$= \frac{1}{3}(k+1)^3 + \frac{1}{3}(-3k + 1 + 6k + 3)$

$= \frac{1}{3}(k+1)^3 + \frac{1}{3}(3k + 4)$

$> \frac{1}{3}(k+1)^3$

[Standard wording]