Induction – Q26 [Practice/E] (18/6/23)

$2^{4n+1} + 3$ is divisible by 5

Solution

[Show that the result is true for $n = 1$]

Now assume that the result is true for $n = k$

Approach 1

so that $2^{4k+1} + 3 = 5M$, where $M \in \mathbb{Z}^+$

To show that the result is then true for $n = k + 1$:

$2^{4k+5} + 3 = 16(5M - 3) + 3$

$= 5(16M - 9)$

(the multiple is positive, as $2^{4k+5} + 3$ is positive)

[Standard wording]

Approach 2

Let $f(k) = 2^{4k+1} + 3$

Then $f(k + 1) - \lambda f(k)$

$= (2^{4k+5} + 3) - \lambda(2^{4k+1} + 3)$

$= 2^{4k+1}(16 - \lambda) + 3(1 - \lambda)$

putting $\lambda = 16$ [or $\lambda = 1$]

$= -45$

so that $f(k + 1) = 16f(k) - 45$

As both terms on the RHS are multiples of 5, it follows that $f(k + 1)$ is a multiple of 5 (and the multiple is positive, as

$2^{4k+5} + 3$ is positive)

[Standard wording]