Induction – Q25 [Practice/E] (18/6/23)

$5^{2n+2} - 24n - 25$ is divisible by 576

Solution

[Show that the result is true for $n = 1$]

Now assume that the result is true for $n = k$

Approach 1

so that $5^{2k+2} - 24k - 25 = 576M$, where $M \in \mathbb{Z}^+$

To show that the result is then true for $n = k + 1$:

$5^{2k+4} - 24(k + 1) - 25 = 25(576M + 24k + 25) - 24k - 49$

$= 576(25M) + 24k(24) + 576$

$= 576(25M + k + 1)$

[Standard wording]

Approach 2

Let $f(k) = 5^{2k+2} - 24k - 25$

Then $f(k + 1) - \lambda f(k)$

$= 5^{2k+4} - 24(k + 1) - 25 - \lambda(5^{2k+2} - 24k - 25)$

$= 5^{2k+2}(25 - \lambda) + k(-24 + 24\lambda) - 49 + 25\lambda$

putting $\lambda = 25$

$= 24(24)k - 49 + 625 = 576(k + 1)$

so that $f(k + 1) = 25f(k) + 576(k + 1)$

As both terms on the RHS are multiples of 576, it follows that $f(k + 1)$ is a multiple of 576

[Standard wording]