# Induction – Q24 [Practice/E] (18/6/23)

$5^{2n} + 12^{n-1}$ is divisible by 13

**Solution**

[Show that the result is true for $n = 1$]

Now assume that the result is true for $n = k$

**Approach 1**

so that $5^{2k} + 12^{k-1} = 13M$ , where $M \in \mathbb{Z}^+$

To show that the result is then true for $n = k + 1$:

$5^{2k+2} + 12^k = 25(13M - 12^{k-1}) + 12^k$

$= 13(25M) + 12^{k-1}(12 - 25)$

$= 13(25M - 12^{k-1})$

(the multiple is positive, as $5^{2k+2} + 12^k$ is positive)

[Standard wording]

**Approach 2**

Let $f(k) = 5^{2k} + 12^{k-1}$

Then $f(k + 1) - \lambda f(k)$

$= 5^{2k+2} + 12^k - \lambda(5^{2k} + 12^{k-1})$

$= 5^{2k}(25 - \lambda) + 12^{k-1}(12 - \lambda)$

putting $\lambda = 25$ [or $\lambda = -1$]

$= -(13)12^{k-1}$

so that $f(k + 1) = 25f(k) - 13(12^{k-1})$

As both terms on the RHS are multiples of 13, it follows that $f(k + 1)$ is a multiple of 13

(the multiple is positive, as $5^{2k+2} + 12^k$ is positive)

[Standard wording]