

Induction – Q2 [Practice/E] (18/6/23)

$$1 \times 4 + 2 \times 5 + 3 \times 6 + \dots + n(n + 3) = \frac{1}{3}n(n + 1)(n + 5)$$

Solution

Result to prove: $\sum_{r=1}^n r(r+3) = \frac{1}{3}n(n+1)(n+5)$

[Show that the result is true for $n = 1$]

Now assume that the result is true for $n = k$, so that

$$\sum_{r=1}^k r(r+3) = \frac{1}{3}k(k+1)(k+5)$$

The target result is

$$\sum_{r=1}^{k+1} r(r+3) = \frac{1}{3}(k+1)(k+2)(k+6)$$

$$\begin{aligned} \text{Then } \sum_{r=1}^{k+1} r(r+3) &= \frac{1}{3}k(k+1)(k+5) + (k+1)(k+4) \\ &= \frac{1}{3}(k+1)\{k(k+5) + 3(k+4)\} = \frac{1}{3}(k+1)(k^2 + 8k + 12) \\ &= \frac{1}{3}(k+1)(k+2)(k+6), \text{ which is the target.} \end{aligned}$$

[Standard wording]