Induction – Q21 [Practice/E] (18/6/23)

$5^n + 12n - 1$ is divisible by 16

Solution

[Show that the result is true for $n = 1$]

Now assume that the result is true for $n = k$

Approach 1

so that $5^k + 12k - 1 = 16M$, where $M \in \mathbb{Z}^+$

To show that the result is then true for $n = k + 1$:

$5^{k+1} + 12(k + 1) - 1 = 5(16M - 12k + 1) + 12k + 11$

$= 16(5M) - 48k + 16$

$= 16(5M - 3k + 1)$

(the multiple is positive, as $5^{k+1} + 12(k + 1) - 1$ is positive)

[Standard wording]

Approach 2

Let $f(k) = 5^k + 12k - 1$

Then $f(k + 1) - \lambda f(k)$

$= 5^{k+1} + 12(k + 1) - 1 - \lambda(5^k + 12k - 1)$

$= 5^k(5 - \lambda) + 12k(1 - \lambda) + 11 + \lambda$

Let $\lambda = 5$, so that $f(k + 1) = 5f(k) - 48k + 16$

As all the terms on the RHS are multiples of 16, it follows that $f(k + 1)$ is a multiple of 16.

[Standard wording]