# **Induction – Q20 [Practice/E]** (18/6/23)

$2^{n+2} + 3^{2n+1}$ is divisible by 7

## Solution

[Show that the result is true for $n = 1$]

Now assume that the result is true for $n = k$

## Approach 1

so that $2^{k+2} + 3^{2k+1} = 7M$, where $M \in \mathbb{Z}^+$

To show that the result is then true for $n = k + 1$:

$2^{(k+1)+2} + 3^{2(k+1)+1} = 2^{k+3} + 3^{2k+3}$

$= 2(2^{k+2}) + 3^{2k+3} = 2(7M - 3^{2k+1}) + 3^{2k+3}$

$= 7(2M) + 3^{2k+1}(-2 + 9)$

$= 7(2M + 3^{2k+1})$

[Standard wording]

## Approach 2

Let $f(k) = 2^{k+2} + 3^{2k+1}$

Then $f(k + 1) - \lambda f(k) = 2^{k+3} + 3^{2k+3} - \lambda(2^{k+2} + 3^{2k+1})$

$= 2^{k+2}(2 - \lambda) + 3^{2k+1}(9 - \lambda)$

Let $\lambda = 2$, so that $f(k + 1) = 2f(k) + 7(3^{2k+1})$

As both terms on the RHS are multiples of 7, it follows that $f(k + 1)$ is a multiple of 7.

[Note that we cannot set $\lambda = 1$ in this example.]

[Standard wording]