Induction – Q18 [Practice/E] (18/6/23)

If $u_{n+1} = \dfrac{u_n}{u_n + 1}$, where $u_n = 1$, suggest a formula for u_n and prove it by induction

Solution

$$u_2 = \frac{u_1}{u_1+1} = \frac{1}{2} \ , u_3 = \frac{u_2}{u_2+1} = \frac{\frac{1}{2}}{\frac{1}{2}+1} = \frac{1}{3} \ , u_4 = \frac{u_3}{u_3+1} = \frac{\frac{1}{3}}{\frac{1}{3}+1} = \frac{1}{4}$$

Suppose that $u_n = \frac{1}{n}$

[Show that the result is true for $n = 1$]

Now assume that the result is true for $n = k$,

so that $u_k = \frac{1}{k}$

Then $u_{k+1} = \frac{u_k}{u_k+1} = \frac{\frac{1}{k}}{\frac{1}{k}+1} = \frac{1}{k+1}$

[Standard wording]