Induction – Q17 [Practice/E] (18/6/23)

If $u_{n+1} = 4n - u_n$, where $u_1 = \frac{1}{2}$, then $u_n = 2n + \frac{1}{2}(-1)^n - 1$

Solution

[Show that the result is true for $n = 1$]

Now assume that the result is true for $n = k$,

so that $u_k = 2k + \frac{1}{2}(-1)^k - 1$

Then $u_{k+1} = 4k - (2k + \frac{1}{2}(-1)^k - 1)$

$= 2k + \frac{1}{2}(-1)^{k+1} + 1$

$= 2(k + 1) + \frac{1}{2}(-1)^{k+1} - 1$

[Standard wording]