

**Induction – Q15 [Practice/E] (18/6/23)**

If  $u_n = 5u_{n-1} - 6u_{n-2}$ , where  $u_0 = -1$  &  $u_1 = -1$ ,

then  $u_n = 3^n - 2^{n+1}$

## Solution

[Show that the result is true for  $n = 1$ ]

[We start with  $n = 1$  because then the method is reliant on using the expression for  $u_{k+1}$ , where  $k = 1$ , and this is defined (as

$5u_k - 6u_{k-1} = 5u_1 - 6u_0$ ), whereas the corresponding expression for  $k = 0$  ( $5u_0 - 6u_{-1}$ ) is not defined.]

Now assume that the result is true for  $n = k$ ,

so that  $u_k = 3^k - 2^{k+1}$

Then  $u_{k+1} = 5u_k - 6u_{k-1} = 5(3^k - 2^{k+1}) - 6(3^{k-1} - 2^k)$

$$= 3^{k-1}(15 - 6) - 2^k(10 - 6)$$

$$= 3^{k+1} - 2^{k+2}$$

$$= 3^{k+1} - 2^{(k+1)+1}$$

[Standard wording]