Induction – Q14 [Practice/E] (18/6/23)

If $u_n = 3u_{n-1} - 2u_{n-2}$, where $u_1 = 1$ & $u_2 = 3$,

then $u_n = 2^n - 1$

Solution

Assume that the result is true for $n = k$ and $n = k + 1$,

so that $u_k = 2^k - 1$ and $u_{k+1} = 2^{k+1} - 1$

Then $u_{k+2} = 3u_{k+1} - 2u_k = 3(2^{k+1} - 1) - 2(2^k - 1)$

$= 2^{k+1}(3 - 1) - 1 = 2^{k+2} - 1$, which is the required result for $n = k + 2$.

Thus if the result is true for $n = k$ and $n = k + 1$, then it is true for $n = k + 2$.

[Show true for $n = 1$ & $n = 2$]

Hence it is true for $n = 3, 4, \ldots$ etc