Induction Overview (18/6/23)

Q1 [Practice/E]

The sum of the 1st n odd numbers is n^2

Q2 [Practice/E]

$1 \times 4 + 2 \times 5 + 3 \times 6 + \cdots + n(n+3) = \frac{1}{3}n(n+1)(n+5)$

Q3 [Practice/E]

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^n} = 1 - \frac{1}{2^n}$

Q4 [Practice/E]

$2 + 4 + 6 + \cdots + 2n = n(n+1)$

Q5 [Practice/E]

$\sum_{r=1}^{n} r(r+1) = \frac{1}{3}n(n+1)(n+2)$

Q6 [Practice/E]

$\sum_{r=1}^{n} r(r+2) = \frac{1}{6}n(n+1)(2n+7)$

Q7 [Practice/E]

$\sum_{r=1}^{n} r(r+1)(r+2) = \frac{1}{4}n(n+1)(n+2)(n+3)$

Q8 [Practice/E]

$\sum_{r=1}^{n} 2^r = 2(2^n - 1)$

Q9 [Practice/E]

$\sum_{r=1}^{n} \frac{1}{(2r-1)(2r+1)} = \frac{n}{2n+1}$

Q10 [Practice/E]

$\sum_{r=1}^{n} \frac{1}{r(r+1)(r+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$

Q11 [Practice/E]

$\sum_{r=1}^{n} r(r!) = (n+1)! - 1$

Q12 [Practice/E]

If $u_n = u_{n-1} + 2$, where $u_1 = 3$, then $u_n = 2n + 1$

Q13 [Practice/E]

If $u_n = 3u_{n-1} + 4$, where $u_1 = 2$, then $u_n = 4(3^{n-1}) - 2$

Q14 [Practice/E]

If $u_n = 3u_{n-1} - 2u_{n-2}$, where $u_1 = 1$ & $u_2 = 3$,

then $u_n = 2^n - 1$

Q15 [Practice/E]

If $u_n = 5u_{n-1} - 6u_{n-2}$, where $u_0 = -1$ & $u_1 = -1$,

then $u_n = 3^n - 2^{n+1}$

Q16 [Practice/E]

If $u_{n+1} = 3u_n - 2^n$, where $u_1 = 5$, then $u_n = 2^n + 3^n$

Q17 [Practice/E]

If $u_{n+1} = 4n - u_n$, where $u_1 = \frac{1}{2}$, then $u_n = 2n + \frac{1}{2}(-1)^n - 1$

Q18 [Practice/E]

If $u_{n+1} = \frac{u_n}{u_n + 1}$, where $u_n = 1$, suggest a formula for u_n and prove it by induction

Q19 [Practice/E]

$7^{2n-1} + 3^{2n}$ is divisible by 8

Q20 [Practice/E]

$2^{n+2} + 3^{2n+1}$ is divisible by 7

Q21 [Practice/E]

$5^n + 12n - 1$ is divisible by 16

Q22 [Practice/E]

$2^{n+1} + 9(13^n)$ is divisible by 11

Q23 [Practice/E]

$13^n + 6^{n-1}$ is divisible by 7

Q24 [Practice/E]

$5^{2n} + 12^{n-1}$ is divisible by 13

Q25 [Practice/E]

$5^{2n+2} - 24n - 25$ is divisible by 576

Q26 [Practice/E]

$2^{4n+1} + 3$ is divisible by 5

Q27 [Practice/M]

$2 + 4 + 6 + \cdots + 2n > n^2$

Q28 [Practice/M]

$\sum_{r=1}^{n} r^2 > \frac{1}{3}n^3$

Q29 [Practice/M]

$\frac{1}{4}n^4 < \sum_{r=1}^{n} r^3 \leq n^4$

Q30 [Practice/M]

The sum of the interior angles of a convex n-sided polygon is $180(n-2)$

Q31 [Practice/E]

If $A = \begin{pmatrix} -1 & -4 \\ 1 & 3 \end{pmatrix}$, then $A^n = \begin{pmatrix} 1-2n & -4n \\ n & 1+2n \end{pmatrix}$

Q32 [Practice/E]

$\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\ldots\left(1-\frac{1}{n^2}\right) = \frac{n+1}{2n}$ for $n \geq 2$