Hyperbolic Functions: Exercises (3 pages; 28/9/19)

(1) (i) Prove, using exponential functions, that

(a) $cosh^2x - sinh^2x = 1$

(b) $sinh2x = 2sinhxcoshx$

(ii) By differentiating the result from (i)(b), obtain an expression for $cosh2x$ in terms of $cosh^2x$ and $sinh^2x$

(2) (a) Find the formula connecting $tanh^2x$ & $sech^2x$?

(b) Find the formula connecting $coth^2x$ & $cosech^2x$?

(3) Show that $artanhx = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) \quad (|x| < 1)$

(4) Differentiation

Find or prove the following:

(i) $\frac{d}{dx}\ tanhx$

(ii) $\frac{d}{dx}arcoshx = \frac{1}{\sqrt{x^2-1}}$

(iii) $\frac{d}{dx}\ artanhx = \frac{1}{1-x^2}$

(iv) $\frac{d}{dx}\ sechx$

(5) Simplify $sinh(cosh^{-1}2)$

(6) Solve the equation $5cosh2x + 3sinhx = 6$,

giving your answers in exact logarithmic form

(7) Show that $arcoshx = \ln(x + \sqrt{x^2 - 1})$

(8) If $x = sinhu$, write $sinh(4u)$ in terms of x

(9) Derive an expression for $arsinh(a)$ in the form lnb

(10) Given that $artanhx = \frac{1}{2}ln\left(\frac{1+x}{1-x}\right)$ and $arcothx = \frac{1}{2}ln\left(\frac{1+x}{x-1}\right)$,

and also that $\frac{d}{dx}(artanhx) = \frac{d}{dx}(arcothx) = \frac{1}{1-x^2}$,

what is wrong with the following reasoning?

$\int \frac{1}{1-x^2}\, dx = artanhx + C = arcothx + C_1$,

so that $artanhx - arcothx = C_2$

But $artanhx - arcothx = \frac{1}{2}ln\left(\frac{\left(\frac{1+x}{1-x}\right)}{\left(\frac{1+x}{x-1}\right)}\right) = \frac{1}{2}ln\left(\frac{x-1}{1-x}\right) = \frac{1}{2}\ln(-1)$,

which isn't defined!

(11) Given that $sinhx = tany$, where $-\frac{\pi}{2} < y < \frac{\pi}{2}$, show that

(a) $tanhx = siny$ (b) $x = \ln(tany + secy)$

(12) What is the domain of $artanh\left(\frac{x}{2}\right)$?

(13) Show that $arcothx = \frac{1}{2}\ln\left(\frac{1+x}{x-1}\right) \quad (|x| > 1)$

(14) (i) Use $artanhx = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$ to show that $\frac{d}{dx}artanhx = \frac{1}{1-x^2}$

(ii) Use $arcothx = \frac{1}{2}\ln\left(\frac{1+x}{x-1}\right)$ to show that $\frac{d}{dx}arcothx = \frac{1}{1-x^2}$ also

(15)(i) Show that $arcothx = artanh\left(\frac{1}{x}\right)$

(ii) Find $f(x)$ such that $arcoshx = arsinh(f(x))$

(16) Given that $\int \frac{1}{\sqrt{x^2-a^2}}dx = arcosh(\frac{x}{a})$, and that

$arcoshx = ln(x + \sqrt{x^2 - 1})$, justify the writing of the integral as
$ln(x + \sqrt{x^2 - a^2})$

(17) Given that $arcoshx = ln(x + \sqrt{x^2 - 1})$, show that if

$cosha = b$ then $a = \ln(b \pm \sqrt{b^2 - 1})$

(18) Sketch the following:

(i) $y = artanhx$ (ii) $y = cothx$ (iii) $y = arcothx$