Hyperbolic Functions – Q8 [Practice/M] (17/6/23)

Show that $artanhx = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) \quad (|x| < 1)$

Solution

If $y = artanhx$, then $tanhy = x \quad (|x| < 1)$

$$\Rightarrow x = \frac{\frac{1}{2}(e^y - e^{-y})}{\frac{1}{2}(e^y + e^{-y})} = \frac{e^{2y} - 1}{e^{2y} + 1}$$

$$\Rightarrow x(e^{2y} + 1) = e^{2y} - 1$$

$$\Rightarrow e^{2y}(x - 1) = -1 - x$$

$$\Rightarrow e^{2y} = \frac{1 + x}{1 - x}$$

$$\Rightarrow y = \frac{1}{2}\ln\left(\frac{1 + x}{1 - x}\right) \quad (|x| < 1)$$