Hyperbolic Functions – Q5 [Practice/E] (17/6/23)

Find or prove the following:

(i) $\frac{d}{dx}\ tanhx$

(ii) $\frac{d}{dx} arcoshx = \frac{1}{\sqrt{x^2-1}}$

(iii) $\frac{d}{dx}\ artanhx = \frac{1}{1-x^2}$

(iv) $\frac{d}{dx}\ sechx$

Solutions

(i) $\frac{d}{dx}\ tanhx = \ \frac{d}{dx}\ \frac{sinhx}{coshx} = \ \frac{coshx.coshx - sinhx.sinhx}{cosh^2x}$

$= \ \frac{1}{cosh^2x} = sech^2x$

(ii) Let $\ y = arcoshx$, so that $\cosh y = x$

Then $\frac{dx}{dy} = sinhy$ and $\frac{dy}{dx} = \frac{1}{\sqrt{cosh^2y - 1}} = \ \frac{1}{\sqrt{x^2-1}}$

(iii) Let $y = artanhx$, so that $tanhy = x$

and $\frac{dx}{dy} = \ sech^2y = 1 - tanh^2y = 1 - x^2$

Hence $\frac{d}{dx}\ artanhx = \ \frac{1}{1-x^2}$

(iv) $\frac{d}{dx}\ sechx = \frac{d}{dx}\ (coshx)^{-1} = (-1)(coshx)^{-2}sinhx$

$= -sech^2x.sinhx$ or $-sechx.tanhx$