Hyperbolic Functions – Q4 [Problem/M](17/6/23)

Given that $\int \frac{1}{\sqrt{x^2-a^2}}\,dx = arcosh(\frac{x}{a})$, and that

$arcoshx = ln\ (x + \sqrt{x^2 - 1})$, justify the writing of the integral as $\ln\,(x + \sqrt{x^2 - a^2})$

Solution

$arcosh\left(\frac{x}{a}\right) = l\,n\left(\frac{x}{a} + \sqrt{\frac{x^2}{a^2} - 1}\right) = \ln\left(\frac{x+\sqrt{x^2-a^2}}{a}\right)$

$= \ln\left(x + \sqrt{x^2 - a^2}\right) - lna$, which only differs from

$\ln\left(x + \sqrt{x^2 - a^2}\right)$ by a constant