Hyperbolic Functions – Q14 [Problem/M](17/6/23)

(i) Show that $arcothx = artanh\left(\frac{1}{x}\right)$

(ii) Find $f(x)$ such that $arcoshx = arsinh(f(x))$

Solution

(i) Let $y = arcothx$, so that $\coth y = x$

$\Rightarrow tanhy = \frac{1}{x}$

$\Rightarrow y = artanh\left(\frac{1}{x}\right)$

(ii) Let $y = arcoshx$, so that $coshy = x$

$\Rightarrow sinhy = \sqrt{x^2 - 1}$

$\Rightarrow y = arsinh\left(\sqrt{x^2 - 1}\right)$; ie $f(x) = \sqrt{x^2 - 1}$