# Hyperbolic Functions – Q12 [Practice/M](17/6/23)

Show that $arcoth x = \frac{1}{2} ln\left(\frac{1+x}{x-1}\right)$ $(|x| > 1)$

## Solution

If $y = arcothx$, then $cothy = x \quad (|x| > 1)$

$$\Rightarrow x = \frac{\frac{1}{2}(e^y + e^{-y})}{\frac{1}{2}(e^y - e^{-y})} = \frac{e^{2y}+1}{e^{2y}-1}$$

$$\Rightarrow x(e^{2y} - 1) = e^{2y} + 1$$

$$\Rightarrow e^{2y}(x - 1) = 1 + x$$

$$\Rightarrow e^{2y} = \frac{1+x}{x-1}$$

$$\Rightarrow y = \frac{1}{2}\ln\left(\frac{1+x}{x-1}\right) \quad (|x| > 1)$$

## Alternative Method

If $artanhx = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) \quad (|x| < 1)$ has been established:

If $y = arcothx$, then $cothy = x$

$\Rightarrow tanhy = \frac{1}{x}$,

and hence $y = artanh\left(\frac{1}{x}\right) = \frac{1}{2}ln\left(\frac{1+\frac{1}{x}}{1-\frac{1}{x}}\right) = \frac{1}{2}ln\left(\frac{x+1}{x-1}\right)$