Hyperbolic Functions – Q1 [Practice/E] (16/6/23)

(i) Prove, using exponential functions, that

(a) $cosh^2x - sinh^2x = 1$

(b) $sinh2x = 2sinhxcoshx$

(ii) By differentiating the result from (i)(b), obtain an expression for $cosh2x$ in terms of $cosh^2x$ and $sinh^2x$

Solution

(i)(a) As $coshx = \frac{1}{2}(e^x + e^{-x})$ & $sinhx = \frac{1}{2}(e^x - e^{-x})$,

$cosh^2x - sinh^2x = (coshx + sinhx)(coshx - sinhx)$

$= e^x.e^{-x} = 1$

(b) $2sinhxcoshx = 2(\frac{1}{2})(e^x - e^{-x})(\frac{1}{2})(e^x + e^{-x})$

$= \frac{1}{2}(e^{2x} - e^{-2x}) = sinh2x$ (by difference of 2 squares)

(ii) Differentiating $sinh2x = 2sinhxcoshx$ gives

$2cosh2x = 2coshxcoshx + 2sinhxsinhx$

$\Rightarrow cosh2x = cosh^2x + sinh^2x$