Hyperbolas - Q5 [Practice/E] (26/5/21)

Show that the equation of the normal to the hyperbola

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point $(acosht, bsinht)$ is

$xasinht + ybcosht = (a^2 + b^2)sinhtcosht$

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Solution

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{bcosht}{asinht}$

so that equation of normal is $y - bsinht = -\frac{asinht}{bcosht}(x - acosht)$

$\Rightarrow bcosht.y - b^2sinhtcosht = -xasinht + a^2sinhtcosht$

$\Rightarrow xasinht + ybcosht = (a^2 + b^2)sinhtcosht$, as required