Hyperbolas Overview (2/7/21)

Q1 [Practice/E]

Show that the equation of the tangent to the hyperbola

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point $(acosht, bsinht)$ is

$yasinht = xbcosht - ab$

Q2 [10 marks]

(i) Given that the tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point $(acosht, bsinht)$ (with equation $yasinht = xbcosht - ab$) meets the asymptotes of the hyperbola at the points P & Q, show that the mid-point of P and Q is $(acosht, bsinht)$. [6 marks]

(ii) In the case where $b = a$, find the area of the triangle OPQ (where O is the Origin). [4 marks]

Q3 [11 marks]

The chord PQ, where P and Q are points on the rectangular hyperbola $xy = c^2$, has gradient 1. Show that the locus of the point of intersection of the tangents from P and Q is the line

$y = -x$.

Q4 [Practice/H]

Use matrices to show that the rectangular hyperbola

$x^2 - y^2 = a^2$ can be obtained by rotating the rectangular hyperbola $xy = c^2$, expressing a^2 in terms of c.

Q5 [Practice/E]

Show that the equation of the normal to the hyperbola

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point $(acosht, bsinht)$ is

$xasinht + ybcosht = (a^2 + b^2)sinhtcosht$

Q6 [9 marks]

Suppose that P is a general point on a rectangular hyperbola and that the tangent at P crosses the x and y axes at A and B respectively. Show that:

(i) $AP = BP$ [7 marks]

(ii) the triangle OAB has a constant area, as P varies [2 marks]