Graphs – Q6 [9 marks] (25/5/21)

Exam Boards

OCR : AL (Year 2)

MEI: AL (Year 2)

AQA: AL (Year 2)

Edx: AL (Year 2)

(i) Find a series of transformations that can be applied to $y = \frac{1}{x}$ to produce $y = \frac{3x-2}{6x-1}$. [6 marks]

(ii) Hence or otherwise, sketch the curve $y = \frac{3x-2}{6x-1}$. [3 marks]

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Solution

(i) $\frac{3x-2}{6x-1} = \frac{3x-\frac{1}{2}-\frac{3}{2}}{6x-1} = \frac{1}{2} - \frac{3}{12}\left(\frac{1}{x-\frac{1}{6}}\right)$ [2 marks]

So a possible series of transformations is:

a translation of $\begin{pmatrix} \frac{1}{6} \\ 0 \end{pmatrix}$, [1 mark]

followed by a stretch of scale factor $\frac{1}{4}$ in the y-direction, [1 mark]

followed by a reflection in the x-axis, [1 mark]

followed by a translation of $\begin{pmatrix} 0 \\ \frac{1}{2} \end{pmatrix}$ [1 mark]

[Note: $\frac{1}{2} - \frac{3}{12}\left(\frac{1}{x-\frac{1}{6}}\right) = \frac{1}{2} - \frac{1}{4x-\frac{2}{3}}$, so an alternative series of transformations is:

a translation of $\begin{pmatrix} \frac{2}{3} \\ 0 \end{pmatrix}$ $[\frac{1}{x} \to \frac{1}{x-\frac{2}{3}}]$ followed by

a stretch of scale factor $\frac{1}{4}$ in the x-direction $[\frac{1}{x-\frac{2}{3}} \to \frac{1}{4x-\frac{2}{3}}]$, followed by a reflection in the x-axis, followed by a translation of $\begin{pmatrix} 0 \\ \frac{1}{2} \end{pmatrix}$.

Alternatively, $\frac{1}{4x-\frac{2}{3}}$ could be obtained instead by a stretch of scale factor $\frac{1}{4}$ in the x-direction $[\frac{1}{x} \to \frac{1}{4x}]$ (or a stretch of scale factor $\frac{1}{4}$ in the y-direction

$[\frac{1}{x} \to \frac{1}{4}(\frac{1}{x})]$), followed by a translation of $\begin{pmatrix} \frac{1}{6} \\ 0 \end{pmatrix}$ $[\frac{1}{4x} \to \frac{1}{4(x-\frac{1}{6})}]$.]

(ii) As an alternative to performing the transformations in (i):

Step 1: $x = 0 \Rightarrow y = 2$; $y = 0 \Rightarrow x = \frac{2}{3}$

Step 2: vertical asymptote when $6x - 1 = 0 \Rightarrow x = \frac{1}{6}$

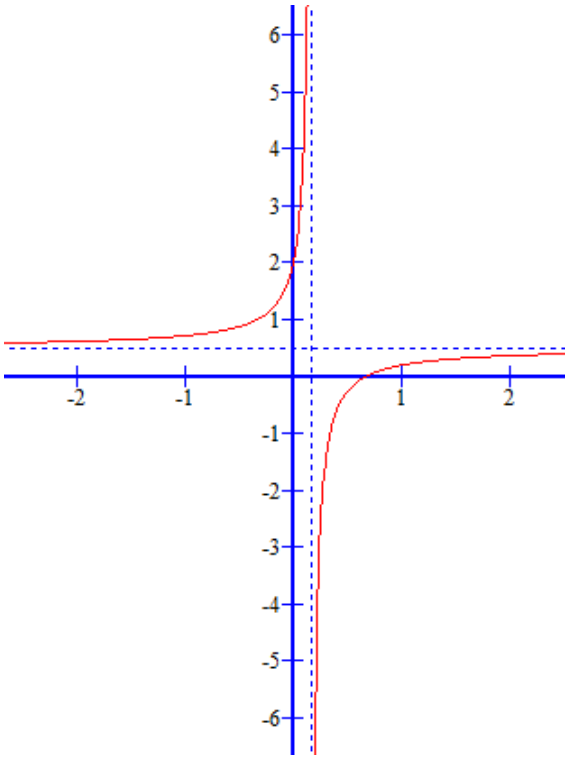
$x = \frac{1}{6} + \delta$ ($\delta > 0$ is small) $\Rightarrow y = \frac{3x-2}{6x-1} = \frac{-}{+}$; ie $y < 0$

$[x = \frac{1}{6} - \delta \Rightarrow y = \frac{-}{-}$; ie $y > 0$]

Step 3: $\lim_{x\to\infty} \frac{3x-2}{6x-1} = \lim_{x\to\infty} \frac{3-\frac{2}{x}}{6-\frac{1}{x}} = \frac{3}{6} = \frac{1}{2}$ (and also as $x \to -\infty$)

Step 4: When $x = 100, y = \frac{298}{599} < \frac{1}{2}$, so that $y \to \frac{1}{2}^-$ as $x \to \infty$

and when $x = -100, y = \frac{-302}{-601} > \frac{1}{2}$, so that $y \to \frac{1}{2}^+$ as $x \to -\infty$



[3 marks]